

Tunable pulse delay and advancement in a coupled nanomechanical resonator-superconducting microwave cavity system

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Abstract

We theoretically study the transmission of a weak probe field under the influence of a strong pump field in a coupled nanomechanical resonator-superconducting microwave cavity system. Using the standard input-output theory, we find that both pulse delay (slow light effect) and advancement (fast light effect) of the probe field can appear in this coupled system provided that we choose the suitable detuning of the pump field from cavity resonance. The magnitude of the delay (advancement) can be tuned continuously by adjusting the power of the pump field. This technique demonstrates great potential in applications including microwave phase shifter and delay line.

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Control of slow and fast light has received a lot of interest in view of its importance in understanding the physical laws that govern how fast or slow a light pulse can be made to propagate and its potential impact on photonic technology [1, 2]. Various techniques have been developed to realize slow light and fast light in atomic vapors and solid state materials. Early studies on slow light have made use of the technique of electromagnetically induced transparency (EIT) in atomic vapors [3] or Bose-Einstein condensate (BEC)[4]. And coherent population oscillation (CPO) is another mechanism that makes slow and fast light possible. Using this technique, Bigelow *et al.* observed a group velocity of 58m/s in a ruby crystal [5] and they also observed both superluminal and ultraslow light propagation in an alexandrite crystal at room temperature [6]. Recently, stimulated Brillouin scattering (SBS) was proposed as an efficient way to realize slow and fast light due to its unique advantages such as arbitrary choice of the a resonant frequency by changing the pump frequency and room-temperature operation. Okawachi *et al.*[7] demonstrated that SBS in a single-mode fiber could be used to induce tunable all-optical slow-light pulse delay as much as 25ns. Observations of both slow and fast light in optical fibers using SBS were also reported [8–10].

On the other hand, nanomechanical resonators (NRs) are currently under intensive exploration owing to their combination of large quality factors ($10^3 \sim 10^5$) and high natural frequencies (MHz \sim GHz) together with the important applications [11, 12], such as high precision displacement detection [13, 14], zeptogram-scale mass sensing [15] and quantum measurements [16]. Quite recently, the analog of electromagnetically induced transparency (EIT) in Fabry-Perot cavity [17] as well as in whispering-gallery-mode (WGM) microresonators [18, 19] has been studied. Alternatively, in the present paper, we theoretically investigate the microwave pulse advancement and delay based on the analog of EIT in the coupled nanomechanical resonator (NR)-superconducting microwave cavity (SMC) system [20]. Much attention has been paid to this coupled system to investigate quantum entanglement, nanomechanical squeezing and cooling of the nanomechanical resonator [21–23]. Here, we propose a scheme to efficiently switch from pulse delay to pulse advancement by simply adjusting the pump-cavity detuning. Our results indicate some potential applications in microwave photonics including microwave phase shifter and delay line [24, 25].

The system under consideration, sketched in figure 1, is a nanomechanical resonator with resonance frequency ω_n and effective mass m capacitively coupled to a superconducting mi-

microwave cavity denoted by the equivalent inductance L and equivalent capacitance C . A strong pump field with frequency ω_p and a weak probe field with frequency ω_r drive the microwave cavity simultaneously. The experiment of such a scheme is usually operated in a dilution refrigerator where the thermomechanical motion is greatly reduced and the transmitted signal is probed through a low-noise high-electron-mobility-transistor (HEMT) microwave amplifier with a homodyne detection scheme. The displacement x of the nanomechanical resonator from its equilibrium position alters the capacitance of the microwave cavity and therefore its resonance frequency. The coupling capacitance can be approximated by $C_0(x) = C_0(1 - x/d)$, where C_0 represents an equilibrium capacitance and d is the equilibrium nanoresonator-cavity separation, thus the coupled cavity has an equivalent capacitance $C_\Sigma = C + C_0$, such that resonance frequency $\omega_c = 1/\sqrt{LC_\Sigma}$.

In a rotating frame at the pump frequency ω_p , the Hamiltonian of the coupled system is given by [20, 21]

$$H = \hbar\Delta_p a^\dagger a + \hbar\omega_n b^\dagger b - \hbar\lambda a^\dagger a Q + i\hbar(E_p a^\dagger - E_p^* a) + i\hbar(E_r a^\dagger e^{-i\delta t} - E_r^* a e^{i\delta t}), \quad (1)$$

Here a^\dagger (a) and b^\dagger (b) are the creation (annihilation) operators of the microwave cavity and nanomechanical resonator, respectively. The first two terms describe the energy of the microwave cavity and the nanomechanical resonator, where $\Delta_p = \omega_c - \omega_p$ is the detuning of the microwave cavity and the pump field. The third term corresponds to the capacitive coupling between the microwave cavity and the nanomechanical resonator, where $\lambda = g\delta x_{zp}$ is the coupling strength between the cavity and the resonator ($g = \frac{\partial\omega_c}{\partial x}$ is the effect of the displacement $x = (b^\dagger + b)\delta x_{zp}$ on the perturbed cavity resonance frequency, $\delta x_{zp} = \sqrt{\frac{\hbar}{2\omega_n m}}$ is the zero-point motion of the nanomechanical resonator) and $Q = b^\dagger + b$ is the phonon amplitude of the resonator. The last two terms give the interaction of the cavity field with the pump field and the probe field, $\delta = \omega_r - \omega_p$ is the detuning of the probe and the pump field, E_p and E_r are, respectively, amplitudes of the pump field and probe field. They are defined by $|E_p| = \sqrt{2P_p\kappa/\hbar\omega_p}$ and $|E_r| = \sqrt{2P_r\kappa/\hbar\omega_r}$, where P_p is the pump power, P_r is the probe power, κ is the linewidth of the microwave cavity.

Let $\langle a \rangle$, $\langle a^\dagger \rangle$, and $\langle Q \rangle$ be the expectation values of the operators a , a^\dagger , and Q , respectively. And in what follows we ignore the quantum properties of a and Q [26–28], the time evolutions of these expectation values can be obtained by employing the Heisenberg equation of motion

and by addition of the damping terms phenomenologically,

$$\frac{d\langle a \rangle}{dt} = -(i\Delta_p + \kappa)\langle a \rangle + i\lambda\langle a \rangle\langle Q \rangle + E_p + E_r e^{-i\delta t}, \quad (2)$$

$$\frac{d^2\langle Q \rangle}{dt^2} + \gamma_n \frac{d\langle Q \rangle}{dt} + \omega_n^2 \langle Q \rangle = 2\omega_n \lambda \langle a^\dagger \rangle \langle a \rangle, \quad (3)$$

where γ_n is the damping rate of the mechanical mode. In order to solve Eq.(2) and Eq.(3), we make the ansatz [29] $\langle a(t) \rangle = a_0 + a_+ e^{-i\delta t} + a_- e^{i\delta t}$, and $\langle Q(t) \rangle = Q_0 + Q_+ e^{-i\delta t} + Q_- e^{i\delta t}$ with the relationship $|a_+|, |a_-| \ll |a_0|$ and $|Q_+|, |Q_-| \ll |Q_0|$. Since the probe field E_r is much weaker than the pump field E_p , we derive the steady state solution of the Eq.(2) and Eq.(3) upon substituting the above ansatz into it and upon working to the lowest order in E_r but to all orders in E_p ,

$$a_+ = \frac{\delta + \Delta_p + i(\kappa + \theta)}{(\delta + i\kappa)^2 + (\theta - i\Delta_p)^2 + \beta} iE_r, \quad (4)$$

where $\eta = \frac{\omega_n^2}{\omega_n^2 - \delta^2 - i\gamma_n \delta}$, $\alpha = \frac{2\lambda^2}{\omega_n^2}$, $\beta = \alpha^2 \eta^2 \omega_n^2 n_p^2$, $\theta = i\alpha \omega_n n_p (\eta + 1)$ and $n_p = |a_0|^2$, approximately equal to the cavity photon occupation, is determined by the equation

$$n_p [\kappa^2 + (\Delta_p - \omega_n \alpha n_p)^2] = |E_p|^2. \quad (5)$$

Using the standard input-output theory [30] $a_{out}(t) = a_{in}(t) - \sqrt{2\kappa}a(t)$, where $a_{out}(t)$ is the output field operator, we obtain

$$\langle a_{out}(t) \rangle = (E_p - \sqrt{2\kappa}a_0)e^{-i\omega t} + (E_r - \sqrt{2\kappa}a_+)e^{-(\delta + \omega_p)t} - \sqrt{2\kappa}a_- e^{i(\delta - \omega_p)t}. \quad (6)$$

The transmission of the probe field, defined by the ratio of the output and input field amplitudes at the probe frequency, is then given by

$$t_p = \frac{E_r - \sqrt{2\kappa}a_+}{E_r} = 1 - \sqrt{2\kappa} \frac{i(\delta + \Delta_p) - (\kappa + \theta)}{(\delta + i\kappa)^2 + (\theta - i\Delta_p)^2 + \beta}. \quad (7)$$

The tunable probe transmission window will modify the propagation dynamics of a probe pulse sent to this coupled system due to the variation of the complex phase picked by its different frequency components. The probe pulse will experiences a group delay τ_g , and this group delay τ_g is defined by $\tau_g = \left. \frac{d\phi}{d\omega_r} \right|_{\omega_c}$, where $\phi(\omega_r) = \arg(t_p(\omega_r))$ is the rapid phase dispersion. The magnitude and phase of the transmitted probe signal could be determined experimentally by measuring the in-phase and quadrature response of the system to the input modulation.

In order to demonstrate our numerical results, we choose a realistic coupled nanomechanical resonator-superconducting microwave cavity system with the parameters as follows [23]: $\omega_c = 2\pi \times 7.5$ GHz, $\omega_n = 2\pi \times 6.3$ MHz, $\kappa = 2\pi \times 600$ kHz, $\lambda = 250$ Hz, and $Q_n = 10^6$, where Q_n is the quality factor of the nanomechanical resonator, and the damping rate γ_n is given by $\frac{\omega_n}{Q_n}$. The system therefore operates in the resolved-sideband regime ($\omega_n > \kappa$) also termed good-cavity limit, which is a prerequisite for cooling of the nanomechanical resonator.

First, we consider the situation where the cavity is driven on its red sideband, i.e., $\Delta_p = \omega_n$, leading to up-conversion of the pump photons to ω_c . Figure 2(a) shows the transmission and phase dispersion of the transmitted probe field as a function of probe-cavity detuning $\Delta_r = \omega_r - \omega_c$ for pump power $P_p = 8nW$ and $\Delta_p = \omega_n$. We find that there is a narrow transparency window with a very steep positive phase dispersion around $\Delta_r = 0$. As a result, there should be slow light effect in this region. This phenomenon is a result of the mechanical analogy of electromagnetically induced transparency (EIT). The simultaneous presence of pump and probe fields generates a radiation force at the beat frequency δ resonant with the mechanical resonant frequency ω_n . The frequency of the pump field ω_p is upshifted to the anti-stokes frequency $\omega_p + \omega_n$, which is degenerate with the probe field. Destructive interference between the anti-stokes field and the probe field can suppress the build-up of an intracavity probe field and result in the narrow transparency window. Here, $\delta = \omega_n$ satisfies the two-photon resonance condition. Furthermore, the effective coupling strength between the microwave cavity and nanoresonator will enlarge with larger number of cavity photon occupation when the pump power increases, and the transparency window will become broader. On the other hand, the displacement x of the nanomechanical resonator from its equilibrium position alters the capacitance of the microwave cavity and therefore its resonance frequency. As a consequence, the effective refractive index seen by the propagating probe field changes and a phase shift is induced. In figure 2(b), we plot the group delay τ_g as a function of the pump power. As the pump power increases, the group delay decreases, and we can get the group delay with the magnitude as much as 0.2 ms at a very low pump power. Therefore, we can tune the probe pulse delay by adjusting the pump power if we set $\Delta_p = \omega_n$.

Furthermore, if the cavity is driven on its blue sideband, i.e., $\Delta_p = -\omega_n$, leading to the down-conversion of the pump photons to ω_c . Similarly, we plot the transmission and phase of the transmitted probe field as a function of the probe-cavity detuning for $P_p = 4nW$ and

$\Delta_p = -\omega_n$ in figure 3(a). We find that there is also a narrow transparency window but with a steep negative phase dispersion around $\Delta_r = 0$, which may result in fast light effect. Here, we have $\delta = -\omega_n$, the frequency of the pump field is downshifted to the stokes frequency $\omega_p - \omega_n$. Destructive interference between the probe field and the stokes field leads to the transparency window. Figure 3(b) shows the group delay τ_g as a function of the pump power for $\delta = \Delta_p = -\omega_n$. The group delay is negative, therefore, we can obtain fast light effect when the cavity is driven on its blue sideband. Time-advanced signals can be used to compensate time delays inevitable in a complex optical-processing network [31]. It's worth noticing that the phenomenon of pulse advancement can be counterintuitive owing to the presence of phenomena where the peak of the output pulse will appear earlier than the peak of the input pulse, its consistency with the principle has been verified experimentally [32]. From figure 2 and figure 3, one can fix the probe field with frequency $\omega_r = \omega_c$, and then scan the pump frequency across the cavity resonance frequency ω_c , one can efficiently switch from probe pulse delay to advancement without appreciable absorption or amplification as the pump detuning Δ_p equals to ω_n or $-\omega_n$.

In conclusion, we have investigated the tunable microwave pulse delay and advancement in a coupled nanomechanical resonator-superconducting microwave cavity system. When the cavity is driven by a strong pump field on its red sideband or blue sideband, the transmitted weak probe field will experience pulse group delay or advancement. Furthermore, the magnitude of the delay or advancement can be tuned by adjusting the power of the pump field in a wide range.

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Figure Captions

Figure 1 (a) Schematic of a nanomechanical resonator capacitively coupled to a microwave cavity in the form of a superconducting coplanar waveguide(denoted by LC circuit) in the presence of a strong pump field ω_p and a weak probe field ω_r . (b) Equivalent circuit.

Figure 2 (a) The normalized magnitude and phase of the cavity transmission as a function of probe-cavity detuning $\Delta_r = \omega_r - \omega_c$ for pump power $P_p = 8nW$ and $\Delta_p = \omega_n$. (b) Group delay as a function of the pump power for $\delta = \Delta_p = \omega_n$. Other parameters are $\omega_c = 2\pi \times 7.5$ GHz, $\kappa = 2\pi \times 600$ kHz, $\lambda = 250$ Hz, $\gamma_n = 40$ Hz and $\omega_n = 2\pi \times 6.3$ MHz.

Figure 3 (a) The normalized magnitude and phase of the cavity transmission as a function of probe-cavity detuning $\Delta_r = \omega_r - \omega_c$ for pump power $P_p = 4nW$ and $\Delta_p = -\omega_n$. (b) Group delay as a function of the pump power for $\delta = \Delta_p = -\omega_n$. Other parameters are $\omega_c = 2\pi \times 7.5$ GHz, $\kappa = 2\pi \times 600$ kHz, $\lambda = 250$ Hz, $\gamma_n = 40$ Hz and $\omega_n = 2\pi \times 6.3$ MHz.

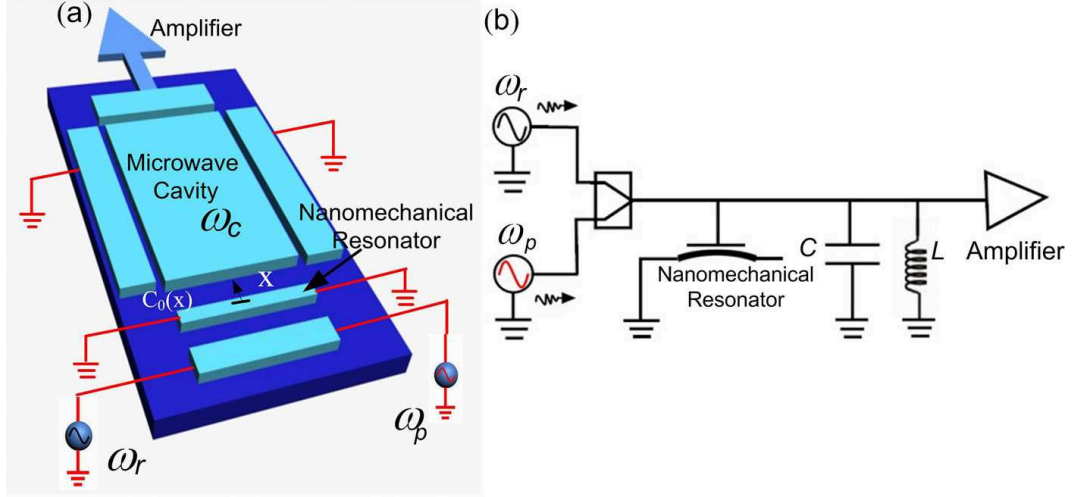


FIG. 1: (a) Schematic of a nanomechanical resonator capacitively coupled to a microwave cavity in the form of a superconducting coplanar waveguide(denoted by LC circuit) in the presence of a strong pump field ω_p and a weak probe field ω_r . (b) Equivalent circuit.

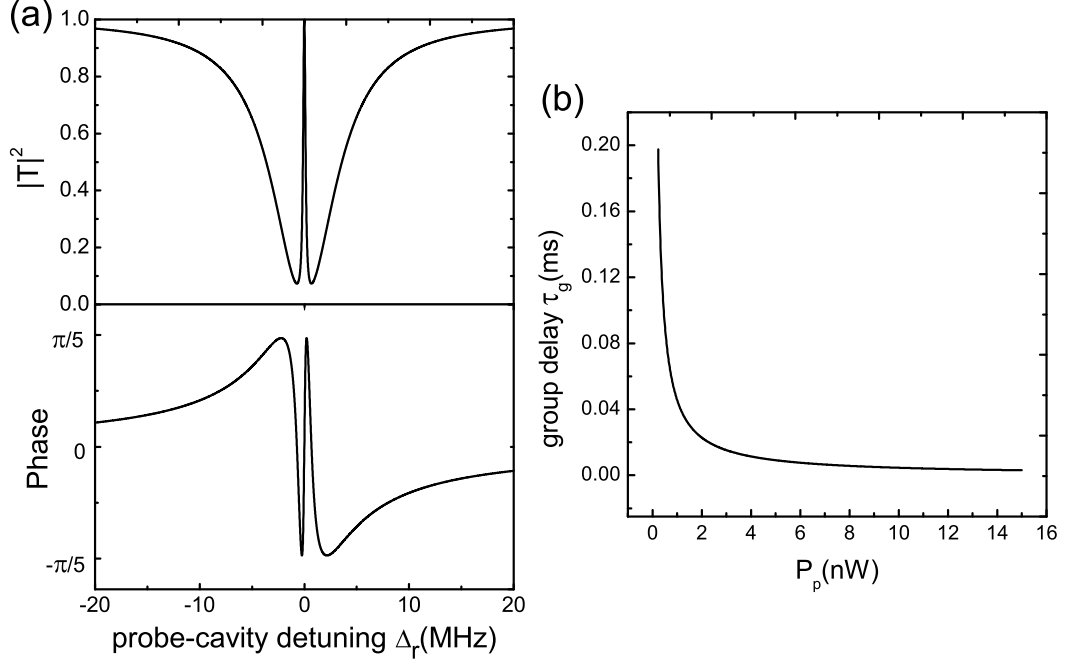


FIG. 2: (a) The normalized magnitude and phase of the cavity transmission as a function of probe-cavity detuning $\Delta_r = \omega_r - \omega_c$ for pump power $P_p = 8 \text{ nW}$ and $\Delta_p = \omega_n$. (b) Group delay as a function of the pump power for $\delta = \Delta_p = \omega_n$. Other parameters are $\omega_c = 2\pi \times 7.5 \text{ GHz}$, $\kappa = 2\pi \times 600 \text{ kHz}$, $\lambda = 250 \text{ Hz}$, $\gamma_n = 40 \text{ Hz}$ and $\omega_n = 2\pi \times 6.3 \text{ MHz}$.

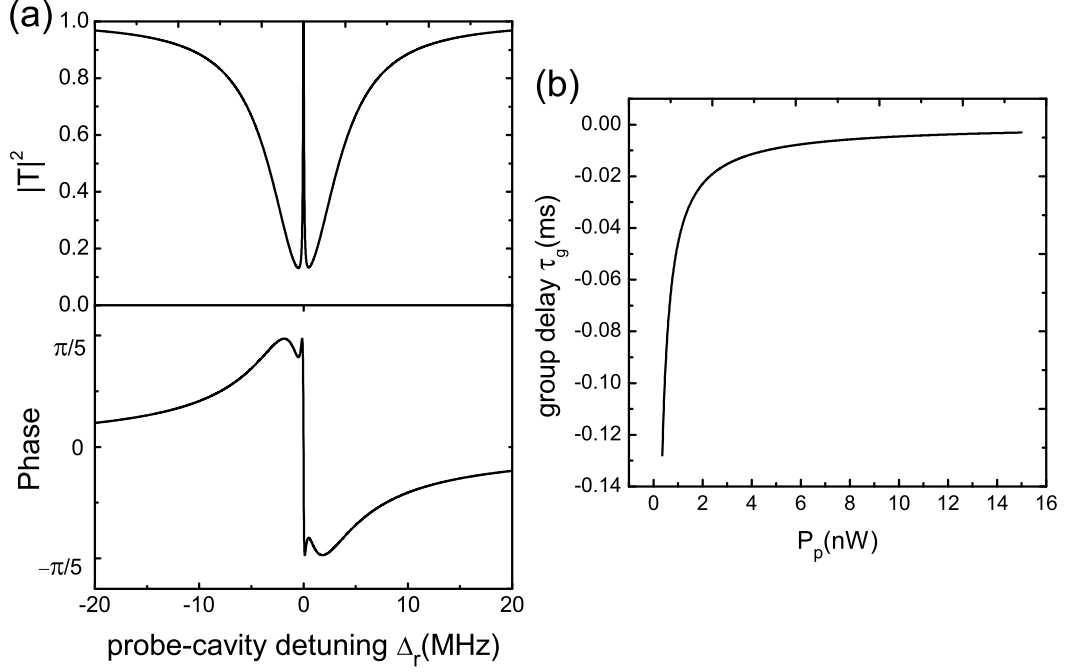


FIG. 3: (a) The normalized magnitude and phase of the cavity transmission as a function of probe-cavity detuning $\Delta_r = \omega_r - \omega_c$ for pump power $P_p = 4\text{ nW}$ and $\Delta_p = -\omega_n$. (b) Group delay as a function of the pump power for $\delta = \Delta_p = -\omega_n$. Other parameters are $\omega_c = 2\pi \times 7.5$ GHz, $\kappa = 2\pi \times 600$ kHz, $\lambda = 250$ Hz, $\gamma_n = 40$ Hz and $\omega_n = 2\pi \times 6.3$ MHz.